Examples of Time Value of Money

Example 1: Assume that you deposit $1,000 today in an account that pays 8 percent interest annually. How much will you have in four years ?

$FV = PV * (1+r)^n = 1{,}000 * 1.08^4 = 1{,}000 * 1.36 = \$1{,}360$

Example 2: Suppose that you have just celebrated your 19th birthday. A rich uncle set up a trust fund for you that will pay you $100,000 when you turn 25. If the relevant discount rate is 11 percent, how much is this fund worth today ?

$PV = FV /(1+r)^6 = 100{,}000/1.11^6 = 100{,}000/1.8704 = \$53{,}460$

Example 3: A first round draft choice quarterback has been signed to a three-year, $10 million contract.

The details provide for an immediate cash bonus of $1 million. The player is to receive $2 million in salary at the end of the first year, $3 million the next, and $4 million at the end of the last year. Assuming a 10 percent discount rate, is this package worth $10 million ?   How mush is it worth ?

$\$1 + 2/1.1 + 3/1.1^2 + 4/1.1^3 = 1 + 1.81 + 2.47 + 3.00 = \$8.30$ million

Example 4 (Annuity Present Value): You are looking into an investment that will pay you $12,000 per year for the next 10 years. If you require a 15 percent return, what is the most you would pay for this investment ?

$PV = A* [(1 - 1/(1+r)^n)]/r = 12{,}000 * [1-(1/1.15)^{10}]/0.15 = 12{,}000*(1-1/4.045)/0.15$

$= 12{,}000 * 0.752/0.15 = 12{,}000 * 5.018 = \$60{,}225$

What is an Annuity?

An **Annuity** is a type of bond that offers a stream of periodic interest payments to the holder until the date of maturity.

$$PV_{Annuity} = \left(\frac{Annuity}{r}\right)\left(1 - \frac{1}{(1 + r)^t}\right)$$

- PV = Present Value
- Annuity = Annuity Payment Per Period ($)
- t = Number of Periods
- r = Yield to Maturity (YTM)

Example 5 (Calculating Present Values):

For each of the following, compute the present value.

Future Value	Years	Interest Rate	Present Value ?
$498	7	13%	$498/(1.13)^7 = \$211.73$
$1,033	13	6%	$1,033/(1.06)^{13} = 484.52$
$14,784	23	4%	$14,784/(1.04)^{23} = 5,998.26$
$898,156	4	31%	$898,152/(1.31)^4 = 304,976.65$

Example 6: An investment has the following cash flows. If the discount rate is 8 percent, what is the present value of these flows ?

Year	Cash Flows	Discount Rate	Present Value
1	$100	8%	$92.59
2	$200	8%	$200/1.08^2 = 200/1.1664 = \$120.19$
3	$700	8%	$300/1.08^3 = 300/1.259 = \$238.28$

Example 7 (Present Value of Multiple Cash Flows): Investment A pays $100 per year for three years. Investment B pays $80 per year for four years. Which of these cash flow streams has the higher PV if the discount rate is 10 percent.

$PV(A) = 100/1.1 + 100/1.1^2 + 100/1.1^3 = 100 + 90.90 + 82.64 = \$273.54$

$PV(B) = 80/1.1 + 80/1.1^2 + 80/1.1^3 + 80/1.11^4 = 74.07 + 66.11 + 60.10 + 54.64 = \$254.92$

Example 8: An investment offers $500 per year for 10 years. If the required return is 10 percent, what is the value of the investment ?
What would be the value be if the them were 30 years ?
Forever ?

1) $500 for 10 years

$PV = 500 * [1-1/1.1^{10}]/0.1 = 500 * [1-1/2.593]/0.1 = 500 * .614/0.1 = \$3070$

2) $500 for 30 years ?
$PV = \$500 * [1-1/1.1^{30}]/0.1 = 500 * [1-0.057]/0.1 = 500 * 9.42 = \$4,710$

3) $500 for forever

$PV = 500/0.1 = \$5,000$