International Management Studies

Lecture 4~5

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I. Time Value of Money and Its Application

1. Time Value of Money

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1) Present Value vs Future Value

The present value formula

Present Value Formula

$$PV = FV / (1 + r)^n$$

PV	=	present value
FV	=	future value
r	=	rate of return
{n}	=	number of periods

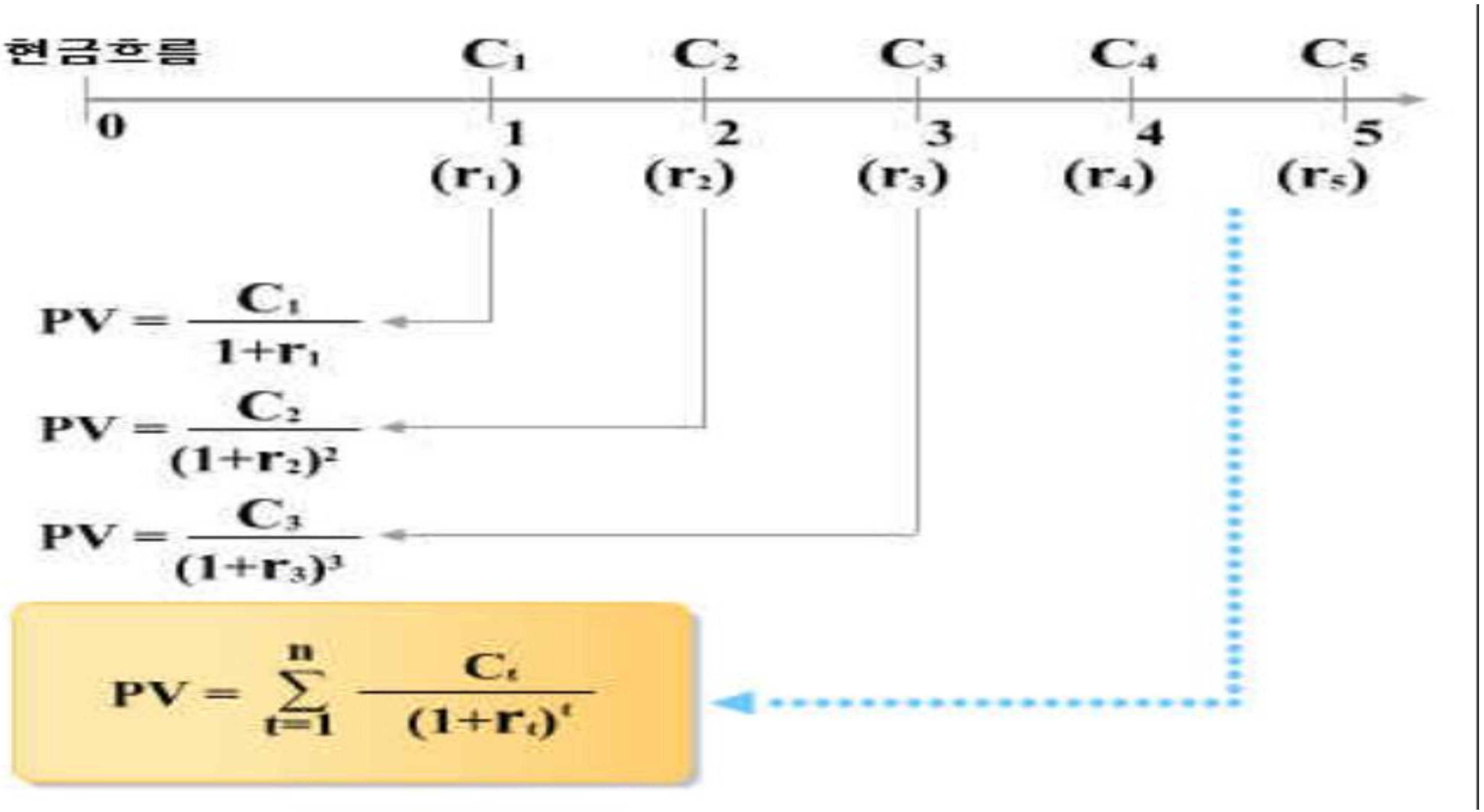
To delve deeper, the formula to calculate Present Value is:

PV = FV / (1 + r)^n

Where:

- PV represents the Present Value.
- FV represents the Future Value.
- r represents the rate of return (or discount rate).
- n represents the number of periods in the future the money is received (or paid).

2) Present value of multiple future cash flows



3) Present value of annuity

Annuity Formula

The future value of an annuity

$$FV = P \times \left(\frac{(1+r)^n - 1}{r}\right)$$

The present value of an annuity

$$PV = P \times \left(\frac{1 - (1+r)^{-n}}{r}\right)$$

Examples Using Annuity Formula

Example 1: Dan was getting $100 for 5 years every year at an interest rate of 5%. Find the future value of this annuity at the end of 5 years? Calculate it by using the annuity formula.

Solution

The future value

Given: $r = 0.05$, 5 years = 5 yearly payments, so $n = 5$, and $P = \$10$C

$FV = P\times((1+r)^n-1) / r$

$FV = \$100 \times ((1+0.05)^5-1) / 0.05$

$FV = 100 \times 55.256$

$FV = \$552.56$

Therefore, the future value of annuity after the end of 5 years is $552.56.

Example 2: If the present value of the annuity is $20,000. Assuming a monthly interest rate of 0.5%, find the value of each payment after every month for 10 years. Calculate it by using the annuity formula.

Solution:

Given:

$r = 0.5\% = 0.005$

$n = 10$ years x 12 months = 120, and PV = $20,000

Using formula for present value

$PV = P \times (1-(1+r)^{-n}) / r$

Or, $P = PV \times (r / (1-(1+r)^{-n}))$

$P = \$20{,}000 \times (0.005 / (1-(1.005)^{-120}))$

$P = \$20{,}000 \times (0.005/ (1-0.54963))$

$P = \$20{,}000 \times 0.011...$

$P = \$220$

Therefore, the value of each payment is $220.

Example 3: Jane won a lottery worth $20,000,000 and has opted for an annuity payment at the end of each year for the next 10 years as a payout option. Determine the amount that Jane will be paid as annuity payment if the constant rate of interest in the market is 5%.

Solution:

Given:

PVA (ordinary) = $20,000,000 (since the annuity to be paid at the end of each year)

$r = 5\%$

$n = 10$ years

Using the Annuity Formula,

$$\text{Annuity} = r * \text{PVA Ordinary} / [1 – (1 + r)^{-n}]$$

$$\text{Annuity} = 5\% \times 20000000 / [1 - (1 + 0.05)^{-10}$$

Annuity = $2,564,102.56

Therefore, Jane will pay an annuity amount of $2,564,102.56

4) Present Value of Perpetuity with Equal Future Cash Flows

PV of Perpetuity

PV of Perpetuity Calculator (Click Here or Scroll Down)

$$PV\ of\ Perpetuity = \frac{D}{r}$$

$PV = Present\ Value$
$D = Dividend\ or\ Coupon\ per\ period$
$r = discount\ rate$

Example of Perpetuity Value Formula

An individual is offered a bond that pays coupon payments of $10 per year and continues for an infinite amount of time. Assuming a 5% discount rate, the formula would be written as

$$\frac{\$10}{.05}$$

After solving, the amount expected to pay for this perpetuity would be $200.

II. Application of Time Value of Money

1. NPV
2. Bond Valuation
3. Stock Valuation

1. NPV (Net Present Value)

What Is Net Present Value (NPV)?

Net present value (NPV) is the difference between the present value of cash inflows and the present value of cash outflows over a period of time. NPV is used in capital budgeting and investment planning to analyze a project's projected profitability.

NPV is the result of calculations that find the current value of a future stream of payments using the proper discount rate. In general, projects with a positive NPV are worth undertaking, while those with a negative NPV are not. [1]

NET PRESENT VALUE (NPV)

$$\text{Net Present Value (NPV)} = -C_0 + \frac{C_1}{(1+r)} + \frac{C_2}{(1+r)^2} + \dots + \frac{C_T}{(1+r)^T}$$

$-C_0$ = Initial Investment

r = Discount Rate

C = Cash Flow

T = Time

The Motley Fool

Image source: The Motley Fool

Net Present Value (NPV) Formula

If there's one cash flow from a project that will be paid one year from now, then the calculation for the NPV of the project is as follows:

$$NPV = \frac{\text{Cash flow}}{(1+i)^t} - \text{initial investment}$$

where:

$i = \text{Required return or discount rate}$

$t = \text{Number of time periods}$

If analyzing a longer-term project with multiple cash flows, then the formula for the NPV of the project is as follows:

$$NPV = \sum_{t=0}^{n} \frac{R_t}{(1+i)^t}$$

where:

$R_t = \text{net cash inflow-outflows during a single period } t$

$i = \text{discount rate or return that could be earned in alt}$

$t = \text{number of time periods}$

How to Calculate NPV Using Excel

In Excel, there is an NPV function that can be used to easily calculate the net present value of a series of cash flows. This is a common tool in financial modeling. The NPV function in Excel is simply "NPV," and the full formula requirement is:

=NPV(discount rate, future cash flow) + initial investment

	A	B	C	D
1				
2				
3		Discount Rate	5%	
4				
5		Initial Investment	$ (500)	
6		Year 1	$ 50	
7		Year 2	$ 75	
8		Year 3	$ 100	
9		Year 4	$ 150	
10		Year 5	$ 250	
11				
12		NPV	$ 21.32	
13				

NPV Example. Excel

How to calculate net present value

Using the data below, let's walk through an example to better understand how to determine a project's NPV.

Time	Cash Flows	Present Value
0	-$15,000.00	-$15,000.00
1	$3,000.00	$2,727.27
2	$3,000.00	$2,479.34
3	$3,000.00	$2,253.94
4	$3,000.00	$2,049.04
5	$3,000.00	$1,862.76
6	$3,000.00	$1,693.42
7	$3,000.00	$1,539.47
8	$3,000.00	$1,399.52
9	$3,000.00	$1,272.29
10	$3,000.00	$1,156.63
Net Present Value at 10%		**$3,433.70**

2. Bond Valuation

$$\text{Bond value} = \sum_{t=1}^{T} \frac{Coupon}{(1+r)^t} + \left[\frac{Face\ Value}{(1+r)^t} \right]$$

Coupon Bond Valuation

Calculating the value of a coupon bond factors in the annual or semi-annual coupon payment and the par value of the bond.

The present value of expected cash flows is added to the present value of the face value of the bond as seen in the following formula:

$$V_{\text{coupons}} = \sum \frac{C}{(1+r)^t}$$

$$V_{\text{face value}} = \frac{F}{(1+r)^T}$$

where:

C = future cash flows, that is, coupon payments

r = discount rate, that is, yield to maturity

F = face value of the bond

t = number of periods

T = time to maturity

For example, let's find the value of a corporate bond with an annual interest rate of 5%, making semi-annual interest payments for two years, after which the bond matures and the principal must be repaid. Assume a YTM of 3%:

- $F = \$1,000$ for corporate bond
- Coupon $rate_{annual} = 5\%$, therefore, Coupon $rate_{semi\text{-}annual} = 5\% / 2 = 2.5\%$
- $C = 2.5\% \times \$1000 = \$25$ per period
- $t = 2$ years $\times 2 = 4$ periods for semi-annual coupon payments
- $T = 4$ periods
- $r =$ YTM of $3\% / 2$ for semi-annual compounding $= 1.5\%$

1. Present value of semi-annual payments $= 25 / (1.015)^1 + 25 / (1.015)^2 + 25 / (1.015)^3 + 25 / (1.015)^4 = 96.36$
2. Present value of face value $= 1000 / (1.015)^4 = 942.18$

Therefore, the value of the bond is $1,038.54.

3. Stock Valuation

Common Stock Valuation

- Common Stock Dividend Valuation Model

$$P_0 = \frac{D_1}{(1+r)^1} + \frac{D_2}{(1+r)^2} + \ldots + \frac{D_\infty}{(1+r)^\infty}$$

- P_0 = Value today of common stock
- D_t = Dividend expected at the end of year t
- r = Required return on common stock

III. Portfolio Theory

What Is the Modern Portfolio Theory (MPT)?

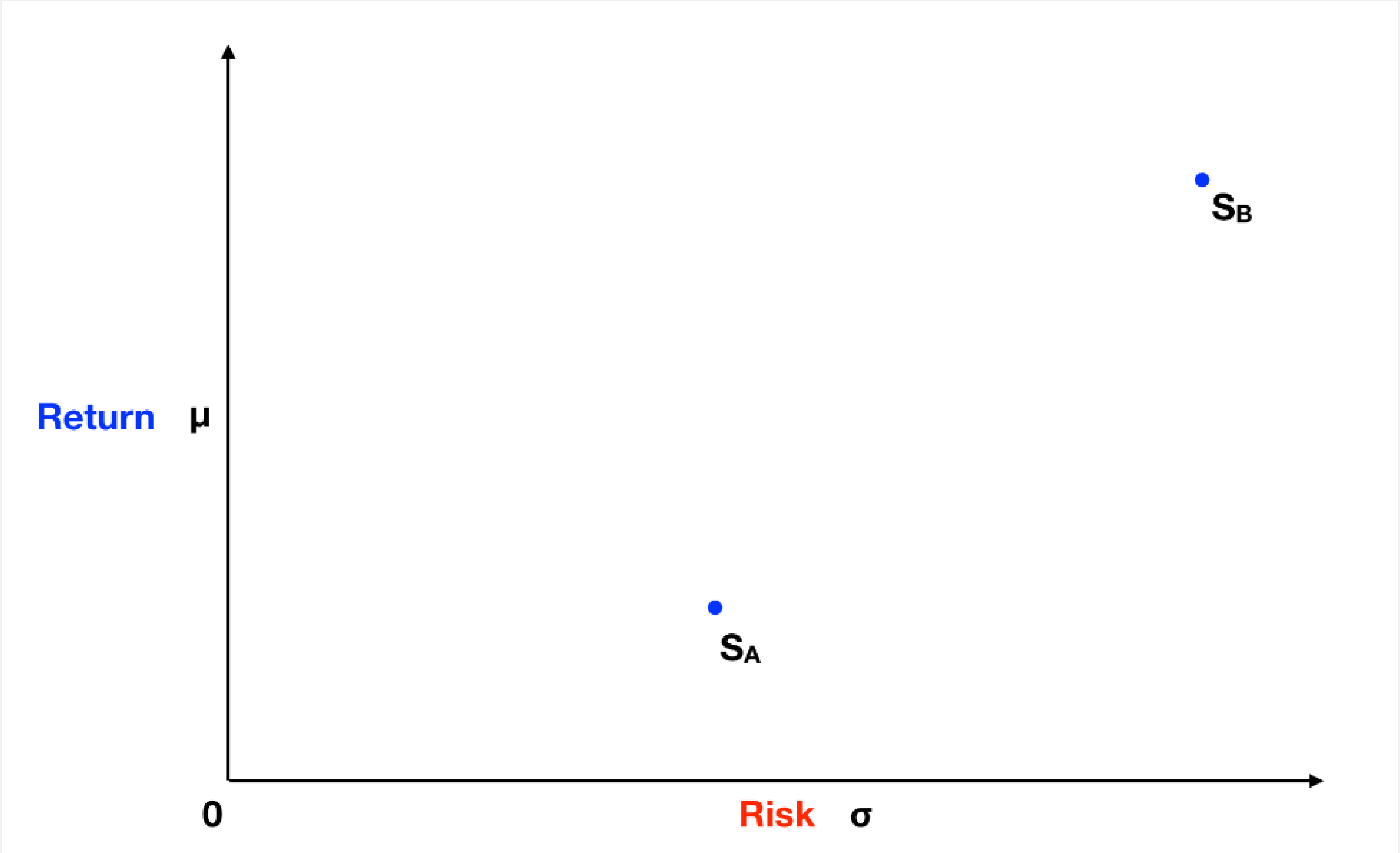
The modern portfolio theory (MPT) is a practical method for selecting investments in order to maximize their overall returns within an acceptable level of risk. This mathematical framework is used to build a portfolio of investments that maximize the amount of expected return for the collective given level of risk.

American economist Harry Markowitz pioneered this theory in his paper "Portfolio Selection," which was published in the Journal of Finance in 1952. [1] He was later awarded a Nobel Prize for his work on modern portfolio theory. [2]

A key component of the MPT theory is diversification. Most investments are either high risk and high return or low risk and low return. Markowitz argued that investors could achieve their best results by choosing an optimal mix of the two based on an assessment of their individual tolerance to risk.

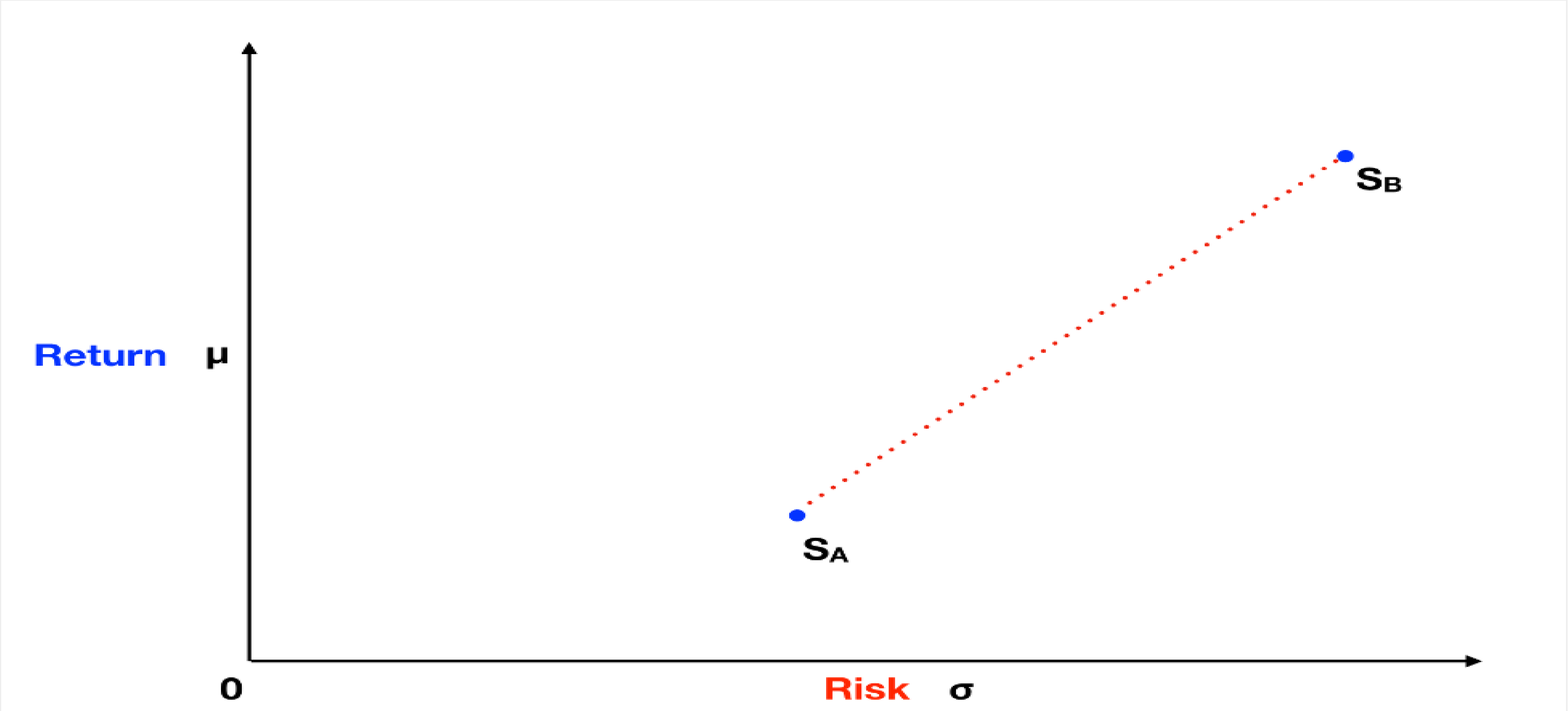
Comparing Assets

Let's start by defining two assets, stock A and stock B, in terms of their expected return and risk, where risk is determined by the stock's volatility in the market. We can then plot and compare these two stocks on the following graph, where the Y axis represents the expected return and the X axis the risk of the stocks.

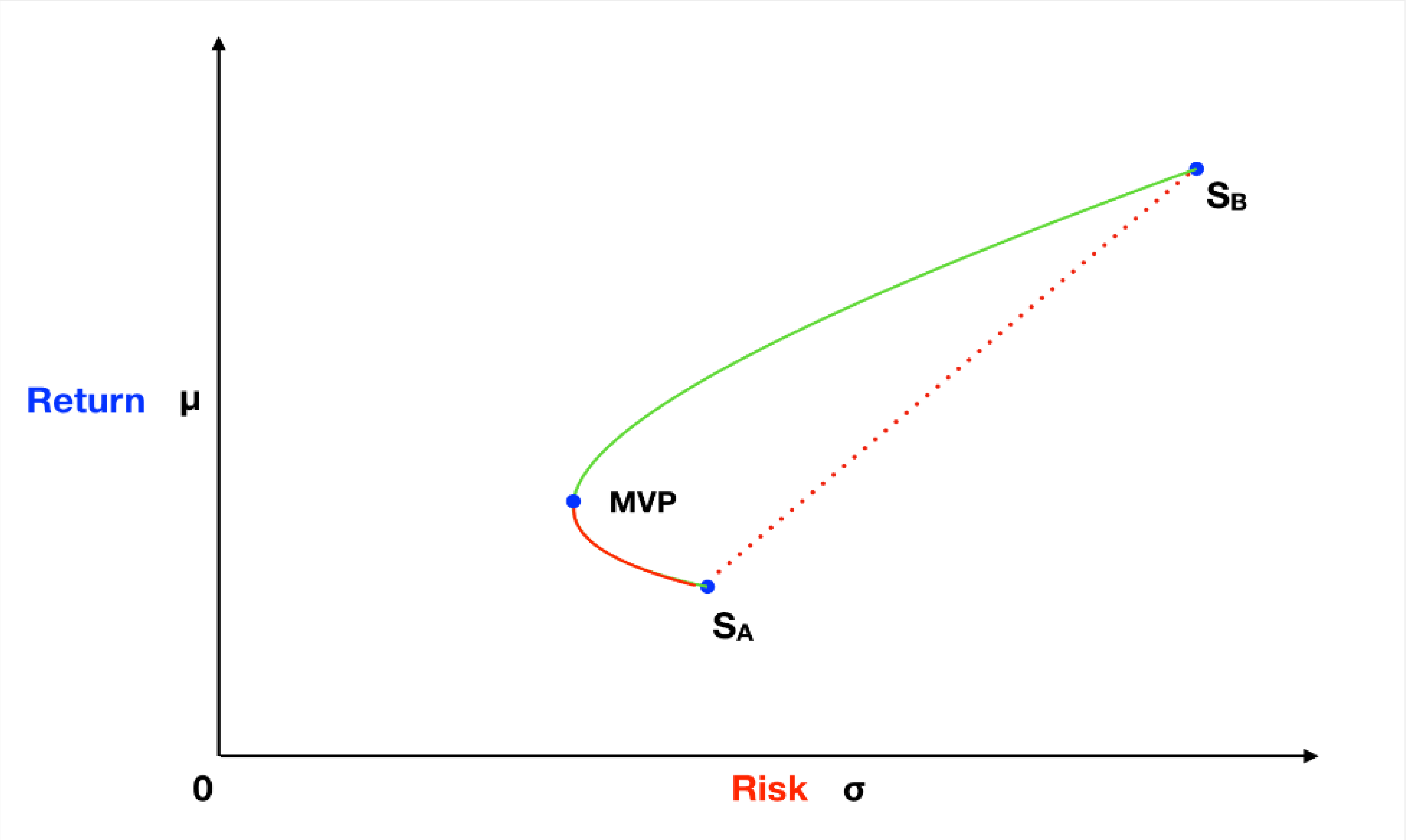


Basic Intuition

We can easily see that stock B has a higher expected return and higher risk than stock A. But can you predict where in this graph is a portfolio with a 50% weight split between these two stocks? Our natural inclination says that such portfolio is half way on a straight line between stock A and stock B. And that a portfolio with 75% weight on stock A and 25% weight on stock B, is one fourth of the way going up that line. The red dotted line on the graph below represents all the possible portfolios between stock A and stock B, as predicted by our basic intuition.



In this case, our intuition is wrong! The line of all possible portfolios of stock A and stock B is not straight, it is curved in the direction of the Y axis, having a hyperbola shape. This is pretty amazing, because it means that we can create portfolios with stock A and stock B, which have less risk than either of these stocks! On the next graph, the blue dot with the label MVP, shows one of these possible portfolios, specifically called the *Minimum Variance Portfolio*.



Correlation Magic

So where is the magic? How is that possible? To solve this puzzle, we need to understand how the portfolio expected return and risk are calculated. I am going to use the two-asset portfolio formula provide on the Wikipedia page for Modern Portfolio Theory. These formulas are easy to deduct with the use of algebra and basic finance knowledge, but I'll spare you the math, so we can focus on the results.

The portfolio expected return calculation is quite simple. It's just a weighted average of the individual stock returns, as illustrated by the following formula:

$$E(R_p) = \omega_A * E(R_A) + \omega_B * E(R_B) = \omega_A * E(R_A) + (1 - \omega_A) * E(R_B)$$

Portfolio Expected Return Formula

Where E(R) is the expected return and ω is the weight allocated to each stock.

This formula is consistent with our initial intuition, that the plot of portfolios created with stock A and stock B would be on a straight line between these two stocks. Now, let's analyze the formula for portfolio variance, which is the square of standard deviation, our chosen measure of risk.

$$\sigma_p^2 = \omega_A^2 * \sigma_A^2 + \omega_B^2 * \sigma_B^2 + 2 * \omega_A * \omega_B * \sigma_A * \sigma_B * \rho_{AB}$$

Portfolio Variance Formula

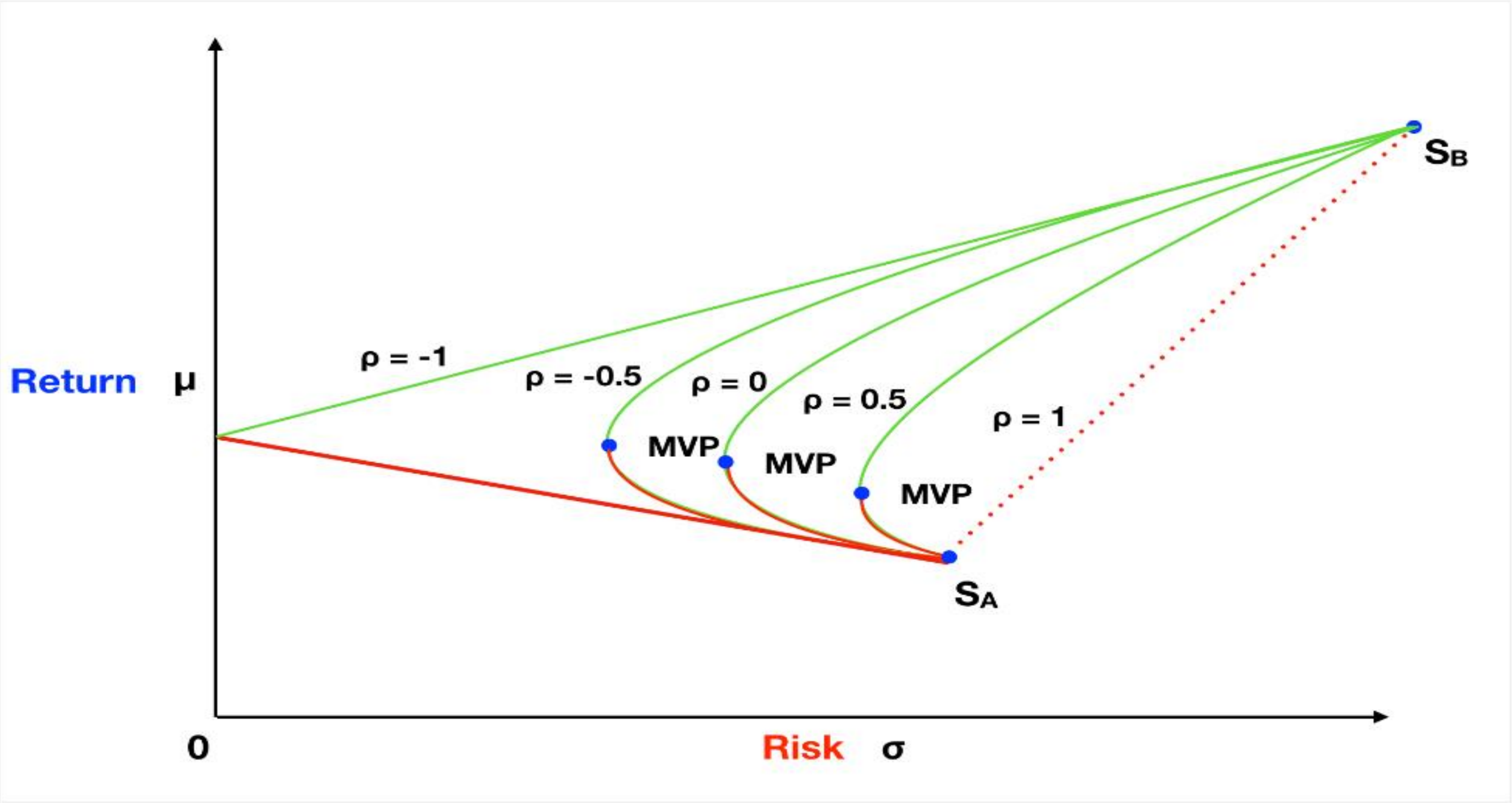
Where σ is the standard deviation of each stock and ρ is the correlation between stocks.

This is where our intuition is failing us. The portfolio variance formula has an additional, and unexpected, component: the correlation between the stocks. Correlation is a number varying between -1 and 1 and describes how the stock returns are related. If the two stocks had a perfect positive correlation, ρ would be equal 1, and the formula would look like this:

$$\sigma_p^2 = \omega_A^2 * \sigma_A^2 + \omega_B^2 * \sigma_B^2 + 2 * \omega_A * \omega_B * \sigma_A * \sigma_B * \rho_{AB} = (\omega_A * \sigma_A + \omega_B * \sigma_B)^2$$

Portfolio Variance Formula → A and B Correlation = 1

This would be coherent with our initial intuition, that all portfolios formed with stock A and stock B would be on the straight line between A and B. But in the market, stocks rarely have perfect positive correlation. Stock A could go up by 1% and stock B go up by 2% in one month, and then on the following month, stock A goes up by 0.5% and stock B goes down by 0.7%. So ρ will probably be a number lower than 1, therefore, reducing the overall portfolio variance. The graph below shows hypothetical hyperbola curves, each using a different correlation, of portfolios created by combining stock A and B. You can easily see that the lower the correlation of these two stocks, the bigger the reduction in portfolio variance, hence, the lower the portfolio risk.



Risk and Return Graph For Two Assets → Varying Correlations

Next: Portfolio Management